Electrical Circuits (2)

Section (5)

Parallel Resonance

Sheet (4)

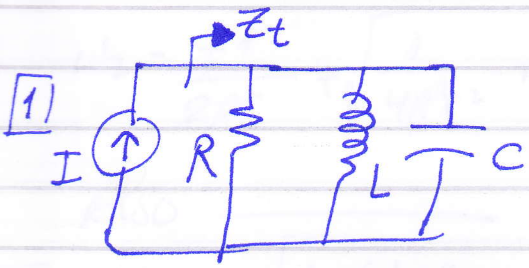
15-17/3/2015

1

Parallel Resonance

Ideal circuit

[1]



[2] $X_L = X_C$ at Resonance

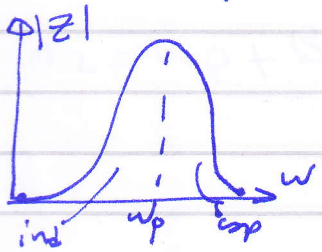
$\therefore \omega_p = \frac{1}{\sqrt{LC}}$ rad/s

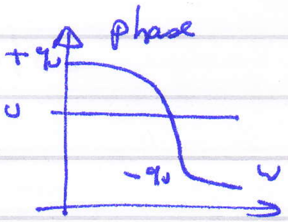
[3] $Y = \frac{1}{Z} = Y_1 + Y_2 + Y_3$

$= \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$

[4] at Resonance $G = \frac{1}{R}$ ← admittance

& V, I in phase.





[5] $Q_p = \frac{R}{X_C} = \frac{R}{X_L} = \frac{\omega_p}{BW}$

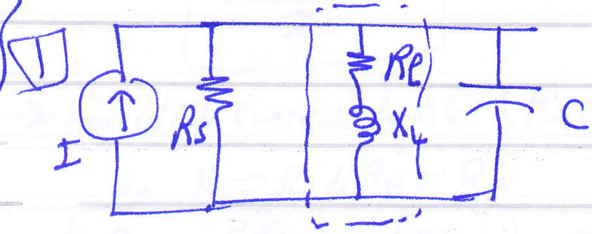
$= R/\omega L = \omega RC$

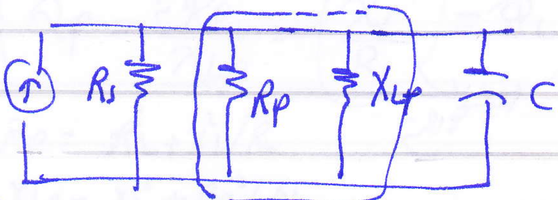
[6] $BW = \omega_2 - \omega_1 = \frac{1}{RC}$ rad/s

$= \omega_p / Q_p$

Practical circuit

[1]





[2] $Z = R_l + jX_L$, $Y = \frac{1}{Z} = \square + j\square$

$$\boxed{R_p = \frac{R_l^2 + X_L^2}{R_L}} \quad \& \quad X_{Lp} = \frac{R_l^2 + X_L^2}{X_L}$$

$= Q_{coil}^2 R_{coil}$, $\approx X_L$

[3] $R = R_s \parallel R_p$

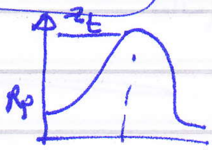
$Y_t = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_{Lp}}\right)$

[4] at Resonance $\frac{1}{X_C} = \frac{1}{X_{Lp}}$

$\therefore f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}}$

$f_0 = f_s$

[5] max impedance freq



$f_m \Rightarrow \frac{dZ_t}{df} = 0$

$f_m = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{1}{4}\frac{R_L^2 C}{L}}$

* Max impedance at $f = 0$ Hz

$X_L = 0$, X_C OC

$\therefore \boxed{Z_t = R_s \parallel R_l \approx R_l}$

$$\boxed{7}\quad \omega_1 = \frac{-1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \ \text{rad/s}$$

$$\omega_2 = \frac{+1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \ \text{rad/s}$$

Also

$$\boxed{8}\quad \omega_1 = \omega_p\sqrt{1+\left(\frac{1}{2Q}\right)^2} - \omega_0/2Q$$

$$\omega_2 = \omega_p\sqrt{1+\left(\frac{1}{2Q}\right)^2} + \omega_0/2Q$$

Note for Midband

$$\omega_1 = \omega_p - B/2$$

$$\omega_2 = \omega_p + B/2$$

$$\boxed{6}\quad Q_p = \frac{R}{X_{Lp}} = \frac{R_s // R_p}{X_{Lp}}$$

$$\boxed{= \frac{R_s // R_p}{X_C}}$$

→ For ideal current source ($R_s \approx \infty$)

$$\therefore R = R_s // R_p \approx R_p$$

$$\therefore \boxed{Q_p = \frac{\approx R_p}{X_{Lp}} = \frac{X_L}{R_L} = Q_L}$$

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→ $R_p = R_L^2 + X_L^2 / R_L$

→ $X_{Lp} = X_L^2 + R_p^2 / X_L$

$$\boxed{7}\quad BW = f_r / Q_p = f_2 - f_1$$

$$= \omega_p / Q_p \qquad \omega_p / Q_p = \frac{1}{RC} \ \text{rad/s}$$

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$

$$\omega_2 = +\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$

$$\boxed{8}\quad Z = \frac{1}{\sqrt{2}} R = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

(3)

Sheet (4)

[1] a- find ω_p, f_p

b- Q

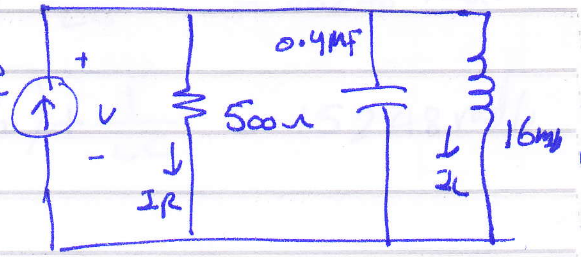
c- voltage across circuit at Resonance

d- I_L, I_R at Res

e- BW rad/s, Hz

f- sketch voltage Res, show voltage at half power points

g- // selectivity curve, show p (watt) / per ω (rad/s)



Sol

a- $\omega_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(16\times10^{-3})(0.4\times10^{-6})}} = 12.5\,\text{Krad/s}$ ✱

$f_p = \omega_p / 2\pi = 1989\,Hz$ ✱

b- $Q_p = \frac{R_p}{X_{Lp}} = \frac{500}{\omega L = (12.5\times10^3)(16\times10^{-3})} = 2.5$ ✱

c- at Resonance $V_C = V_L = V_R$ and so $V = IR$

$= (3.6\times10^{-3})(500) = 1.8\,V\angle0$ ✱

d- $I_L = \frac{V}{X_L} = \frac{1.8\angle0}{\omega L\angle90} = \frac{1.8\angle0}{(12.5\times10^3)(16\times10^{-3})\angle90} = 9\,mA\angle-90$ ✱

$I_R = \frac{V}{R} = \frac{1.8\angle0}{500} = 3.6\,mA\angle0$ ✱

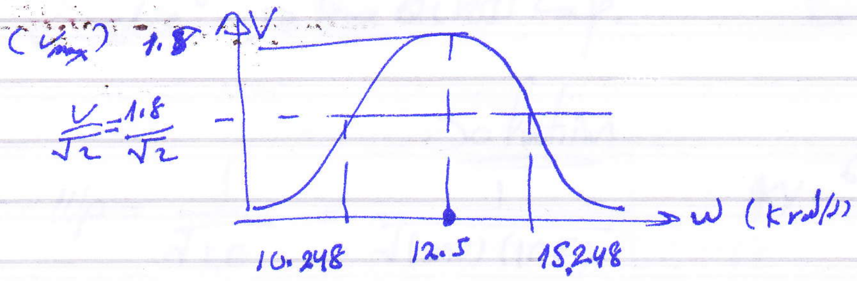
e- $BW\,(rad/s) = \frac{\omega_p}{Q_p} = \frac{12.5\,\text{Krad/s}}{2.5} = 5\,\text{Krad/s}$

$BW\,(Hz) = \frac{BW\,(rad/s)}{2\pi} = 795.8\,HZ$

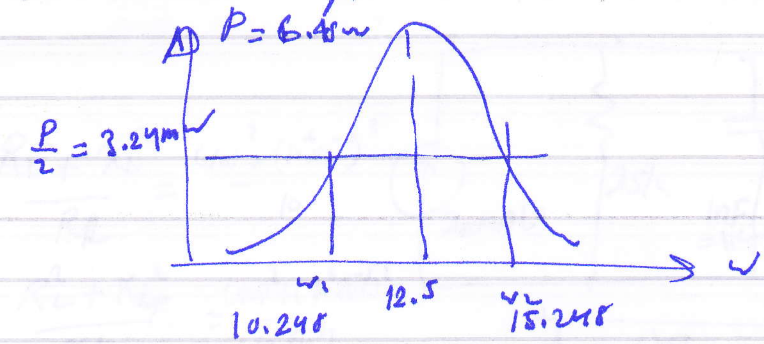
(4)

f- $\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{4R^2C^2}\right) + \frac{1}{LC}} = 10248$ rad/s

$\omega_2 = +\frac{1}{2RC} + \sqrt{\left(\frac{1}{4R^2C^2}\right) + \frac{1}{LC}} = 15248$ rad/s



g- Power $= P = V^2/R = (1.8)^2/500 = 6.48$ mW



end of Prob. (1)

5

Prob 2

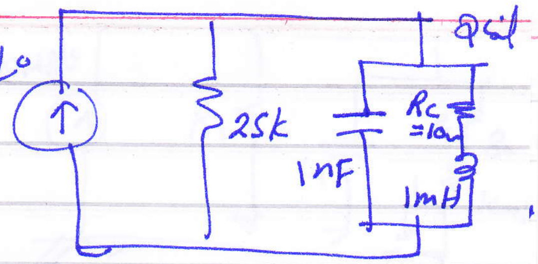
a- ω_p

b- Q of coil at res.

c- sketch equivalent parallel ckt

d- Q of ckt at Res.

e- solve voltage across cap.

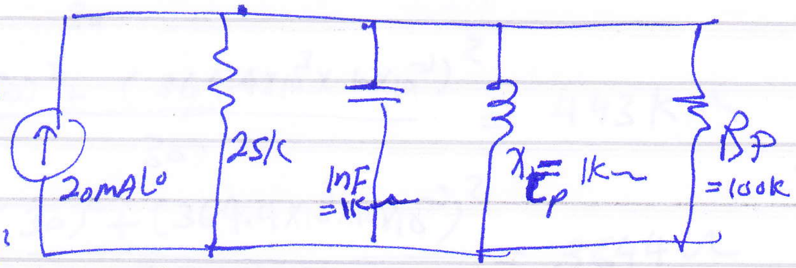


Solution

a- $\omega_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1mH)(1nF)}} = 1\times10^6$ rad/s

b- $Q_{coil} = \frac{\omega L}{R_{coil}} = \frac{(1\times10^6)(1\times10^{-3})}{10} = 100$

c-



$R_p = \frac{R_L^2 + X_L^2}{R_L} = \frac{(10)^2 + (10^6\times10^{-3})^2}{10}$

$X_{Lp} = \frac{R_L^2 + X_L^2}{X_L} = \frac{(10)^2+(10^6\times10^{-3})^2}{(10^6\times10^{-3})}$

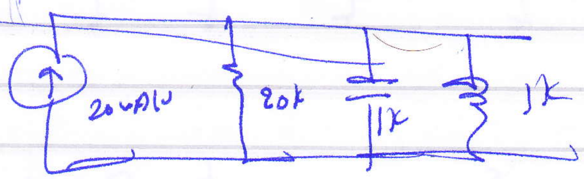
$R_p = 100k\Omega$, $X_L = 1k\Omega$

$\Rightarrow R_{eq} = R_1 // R_p = 20k\Omega$

If Q > 10

$R_p = Q_{coil}^2 R_{coil}$

$X_{Lp} = X_L = \omega L$



d- $Q_p = \frac{R_{eq}}{X_L} = \frac{20K}{1K} = 20$

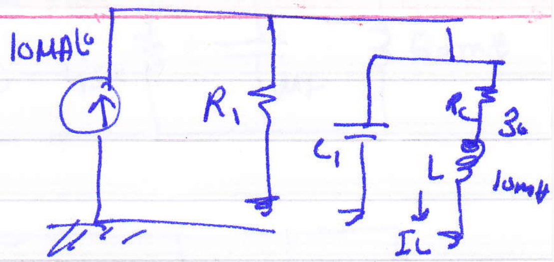
e- at Res $V_C = I R_{eq} = (20mA\angle 0)(20K) = 400V\angle 0$

[illegible]

6

Prob

$\boxed{3}$ (a) Req R_1, C
(b) solve for curnt $\underline{\underline{I_L}}$



Sol

$$\left[\begin{array}{l} Q_p = \dfrac{f_p}{BW} = \dfrac{58k}{1k} = 58 \\ \omega_p = 2\pi f = 2\pi \times 58k = 364.4\,krad/s \\ C = \dfrac{1}{\omega_p^2 L} = \dfrac{1}{(364.4\times10^3)(10\times10^{-3})} = \boxed{753pF} \end{array}\right.$$

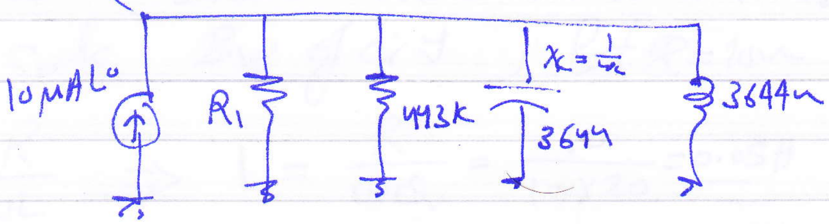
$L = 10mH$
$R_{coil} = 30$
$BW = 1kHz$
$f_p = 58kHz$

$$Q_{coil} = \frac{\omega_p L}{R_{coil}} = \frac{(364.4)\times10^3 \times(10\times10^{-3})}{30} = 121.5$$

$$R_p = \frac{R_{coil}^2 + X_L^2}{R_{coil}} = \frac{(30)^2 + (364.4\times10^3\times10\times10^{-3})^2}{30} = 443k\Omega$$

$$X_{Lp} = \frac{R_{coil}^2 + X_L^2}{X_L} = \frac{(30)^2+(364.4\times10^3\times10\times10^{-3})^2}{(364.4\times10^3\times10\times10^{-3})} = 3644\,\Omega$$

While



$$Q_{sys} = \frac{R}{X_c} = \frac{R}{X_L}$$

$$58 = \frac{R}{3644}$$

$$\therefore R = 211k\Omega$$

$$R = R_1 \,//\, 443k \quad \therefore \frac{443 R_1 \times 10^3}{10^3\times443 + R_1} = R = 211\times10^3$$

$$\therefore R_1 = 405k\Omega$$

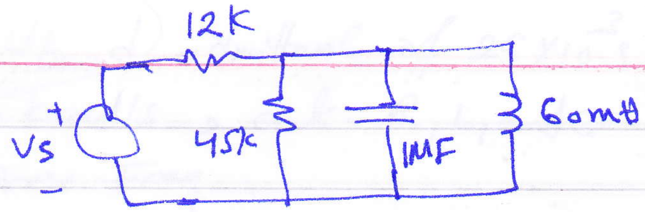
(b) $V = IR = (10mA)(211k\Omega) = 2.11V\angle 0$

Trick $I_L = \dfrac{V}{R_{coil} + jX_L} = \dfrac{2.11}{30 + j3644} = 549\,\mu A\angle{-89.5^\circ}$

[illegible]

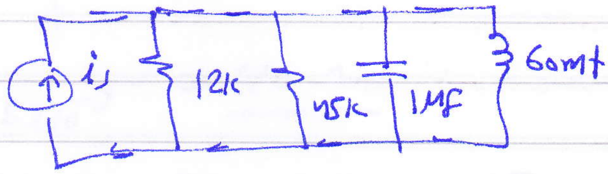
Prob
7

4 let $V_s = 20\cos(\omega t)$, find ω_0, Q, and B assecenby capacitor.



Sol/

$$i_s = \frac{V_s}{12k} = \frac{20\cos\omega t}{12k}$$



& $R = 12k // 45k = 9.47\,k\Omega$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{60\times10^{-3}\times1\times10^{-6}}} = 4.08\ krad/s$$

$$B = \frac{1}{RC} = \frac{1}{(9.47k)(1MF)} = 105.55\ rad/s$$

$$Q = \frac{\omega_0}{B} = \frac{4082k}{105.55k} = 38.67$$

Prob

5 Design a parallel Resonance RLC with $\omega_0 = 10\ rad/s$ & $Q = 20$, calc BW of cit, let $R = 10\Omega$

Sol/ $\rightarrow Q_p = \frac{R}{\omega L} \Rightarrow L = \frac{R}{\omega Q} = \frac{10}{10\times20} = \underline{0.05H}$

$\rightarrow \omega = \frac{1}{\sqrt{LC}}$ or $\omega^2 = \frac{1}{(LC)}$ or $C = \frac{1}{\omega^2 L} = \underline{0.2F}$

$\rightarrow B = \frac{1}{RC} = \frac{1}{10\times0.2} = \underline{0.5\ rad/s}$

6 Parallel RLC, has a midband admittance of 25×10^{-3} S, $Q = 80$, $\omega_0 = 200$ krad/s, calc. R, L, and C. Find BW and half power Freq.

Sol

$\rightarrow Y = \frac{1}{R} \quad \therefore R = \frac{1}{Y} = \frac{1}{25\times10^{-3}} = \underline{40\,\Omega}$

$\rightarrow Q_p = \omega RC = (200\times10^{3})(40)(C) = 80 \Rightarrow \underline{C = 10\,\mu F}$

$\rightarrow \omega = \frac{1}{\sqrt{LC}} \rightarrow \omega^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega^2 C} \Rightarrow \underline{L = 2.5\,\mu H}$

$\rightarrow B = \frac{\omega_p}{Q} = \frac{200k}{80} = 2.5$ krad/s

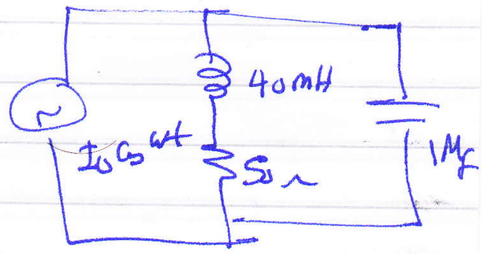
$\rightarrow \omega_1 = \omega_p - B/2 = 200 - 1.25 = 198.75$ krad (Midband)

$\omega_2 = \omega_p + B/2 = 200 + 1.25 = 201.25$ krad/s

ممكن استخدام القانون الطويل ولكن هنا قال Midband

7 For tank circit shown, Find resonance freq.

80



$Y = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_t}$

$= \frac{1}{1/j\omega C} + \frac{1}{R + j\omega L} = Y$

$\therefore Y = j\omega C + \frac{1}{R + j\omega L} = \cancel{\frac{1 + j\omega RC - \omega^2 LC}{R + j\omega L}}$

$Y = j\omega C + \frac{R - j\omega L}{R^2 + (\omega L)^2} = \frac{R}{R^2 + (\omega L)^2} + j\left(\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right)$

at Resonance Imaginary of $Y = 0$

$\cancel{\omega} C = \frac{\cancel{\omega} L}{R^2 + \omega^2 L^2} \quad \therefore R^2 + \omega^2 L^2 = L/C$

$\therefore \omega^2 = \frac{L/C - R^2}{L^2}$ or $\omega^2 = \frac{L/C - R^2}{L^2} = \frac{1}{LC} - \frac{R^2}{L^2}$

$\therefore \omega = \sqrt{\frac{1}{LC} - R^2/L^2} = 4841$ rad/s